Radio Transmissions

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Introduction

This report aimed at giving a shortcut on radio transmissions for all Italian Secondary Teachers of Physics. Such an idea came out of once we realized the huge interest wireless communications system awaken in our students.

The topics herewith discussed can be supplemented by others reported in almost all school books.

This document is divided in the following paragraphs:

Power flux density

It introduces the power flux density concept, giving us the calculation procedures in the presence of a transmitting isotropic antenna. Then the effects in case of a real antenna are discussed. The paragraph also introduces the concept of gain.

> Requirements: the conservation of energy principle, the power concept, the area of a spherical surface, the waves.
>
> Analysis: elements of optics and electromagnetism, antenna typologies.

Signal reception

By receiving antenna reception we mean the “capture” of power distributed on a spherical area, by a little absorbing area. The concept of effective area of a receiving antenna is introduced together with some receiver sensitivity examples.

> Requirements: the concept of power.
>
> Analysis: radio, television, mobile wireless systems.

Antenna reversibility

The paragraph shows that a transmitting antenna can also work as a receiving antenna and viceversa. The relation between the antenna gain and its effective area is also treated.

> Requirements: frequency, wavelength.
>
> Analysis: antenna typologies.

Power flux density and electromagnetic fields

This paragraph illustrates the relations between the power density and the root mean square values of electric and magnetic fields.

The topic has been introduced to better understand radio transmissions as well as the so topical electromagnetic pollution. This paragraph should be treated after the students have learnt both the electromagnetic waves and the root mean square concept, usually explained within the sinusoidal electric quantity area.

> Requirements: electric field, magnetic field, electromagnetic waves and root mean square concept.
>
> Analysis: waves in optic, electromagnetic pollution, operations with vectors and interference.

Receiving and transmitting systems

This paragraph shows the applicatory problems relating to the concepts already reported. This section help the reader to analyze the performances of a basic transmission system in terms of energetic transfer. Students should be taught waves in optic main concepts before facing this part.

> Requirements: elements of waves in optic, reflection, diffraction, interference, Huygens principle.
>
> Analysis: diffraction by fessure, radiocommunications engineering, Poynting vector.

Please note that both in the title of our document and in its content we always refer to the transmission concept not to the communications one. The topics hereunder illustrated let us formulate an energy budget in a radio communications system, without any indication on *how* information is associated to a radio signal. This aspects, closely related to modulations, is treated in our secondary school physic.
I would appreciate my colleagues could help me to enlarge this module with useful further information by mailing any comments and proposals to: iscra@iscra.net .
Any development on this theme will be found at http://www.calvino.ge.it.

Power flux density

Let us consider a small bulb radiating a power P uniformly in the space. Let us consider then a sphere of radius r having as a centre the bulb (Figure 1). The area A of the sphere surface is uniformly crossed by the power P. We can define the power flux density, hereunder S, as the ratio between power P crossing the sphere surface and its area A:

$$S = \frac{P}{A}$$

According to the International System measurements the power density is measured in W/m^2.

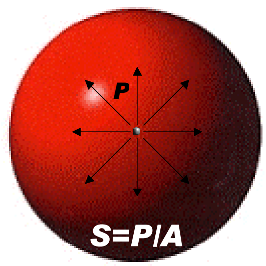

Figure 1. The power radiated by the bulb is distributed on a spherical surface

It's easy understand why the above defined quantity is called power density: a theoretical absorbing body, covering a little area a of the spere, absorbs a power $P_a = aS$, so much higher as the value of S is, that is to say it is proportional to the radiation intensity. The light radiated by a bulb is a kind of electromagnetic radiation of an extremely high frequency (10^{14} Hz). What happens if we use a transmitting antenna instead of the bulb? Just like the bulb, the antenna radiates electromagnetic energy, which differs from the bulb's one as it has a lower frequency (from 10^4 Hz to 10^{10} Hz), and it is coherent (as a laser light). Besides the antenna is unlike to radiate a uniformly power. However, overlooking this characteristic, that is taking into consideration an isotropic antenna (which radiates uniformly), it will exactly work a bulb. It's clear that, being the antenna by far higher than a bulb, the sphere will have a very remarkable radius so that the source located in its centre can be considered as a point.
Knowing that the surface area of a sphere of radius r is $A = 4\pi r^2$, at a distance r from an isotropic antenna the power density is:

$$S = \frac{P}{4\pi r^2}$$

As already stated a real antenna isn't isotropic, but concentrates the radiated energy towards some well defined directions. This is a very useful property as in many cases one or many receivers are located in a fixed position and it's better concentrate towards them the transmitted power. Let's take as example a geostationary satellite, positioned on a circular orbit at the height of 36000 kms on the equator. From that position a whole continent is "seen" under a very small angle and it is therefore necessary to install a very directional antenna on the satellite, being useless transmitting energy towards other directions. The capability of an antenna to concentrate its radiated energy towards a dedicated direction can be quantified with a unit of measurement called *antenna gain*. The gain, thereafter called G_t, is defined so that each point on a sphere having as a centre the transmitting antenna is subject to a radiation intensity:

$$S = \frac{PG_t}{4\pi r^2}$$

in other words, it is as if the antenna, compared with the point, were still isotropic and transmitted a power $P_i = PG_t$. P_i is called *Equivalent Isotropic Radiated Power* (EIRP), very used in the electromagnetic field.

Example

An antenna, whose gain is $G_t = 20$, radiates a power $P = 5$ W. Determine the EIRP and the density power values at a distance of 10 kms.

R: We obtain EIRP = $PG_t = 5\times20 = 100$ W,

$$S = \frac{PG_t}{4\pi r^2} = \frac{EIRP}{4\pi r^2} = \frac{100}{4\pi 10000^2} = 79.6\times10^{-9}\ \text{W/m}^2$$

The signal reception

How does a receiving antenna work? The most evident example is given by the dish antenna, used for the reception of television signal transmitted by satellites. The power intercepted by the parabolic disc, is reflected and concentrated towards the receiver located on the focus of the paraboloid.

An objective analysis can be formulated by considering the following hypothesis:

1. At a long distance from the transmitting antenna, the hypothetical sphere, on which power is distributed, can be locally approximated as a plain, making electromagnetic waves turn into plain waves
2. The parabolic reflector has its cross section orthogonal to the plain waves propagation direction, which totally reflects acting like an ideal mirror.

If these hypothesis occur, the power, captured by the reflector and transmitted towards the receiver, indicated as P_r, is obtained by multiplying the radiation intensity S for the area of the dish section, thereafter called A_r. Therefore:

$$P_r = SA_r$$

The first hypothesis is almost sure to occur while as far the second one is concerned we can state that there no ideal reflector: a small part of the captured power is lost; furthermore geometric optical rules are not properly valid to interpret reflection phenomena due to diffraction. Finally, there's a wide range of antennas for which the signal reception doesn't take place through an intuitive capturing phenomena. Within this last condition we can allocate all the wired antennas, such as the well known dipole. It always stands that, being the transmitting antenna position unchanged in respect of the receiving one, the power received by the second is proportional to the power flux density, i.e.:

$$P_r = kS$$

where k is a coefficient depending on the type of antenna (as well as it is orientated towards the transmitter) and it has the unit of measurement of an area (being P measured in W and S in W/m^2); therefore the coefficient k is called *antenna effective area,* and it will be thereafter indicated as A_{eqr}.

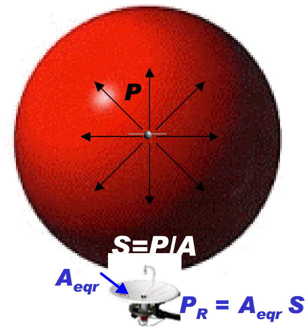

Figure 2. A parabolic reflector captures power by intercepting the sphere

Now we can calculate the power received by a receiving antenna, whose equivalent area is A_{eqr}, located at a distance r from a transmitting antenna of gain G_t that is fed by a power P. We obtain:

$$P_r = A_{eqr} S = A_{eqr} \frac{PG_t}{4\pi r^2}$$

Once we get the fundamental link existing between the transmitted power and the one received in a radio link, we have to evaluate the P_r value to make a receiver work correctly. Progress in technology has allowed the designing of receivers with high sensitivity: a GSM phone can perfectly receive a signal having a power of 10^{-12} W; a car audio reproduces a good quality music with power of 10^{-11} W; a satellite receiver needs about 10^{-9} W.

Example

A car audio is directly connected to an antenna having an equivalent area $A_{eqr} = 0.1$ m^2 and must receive a power of $P_R = 10^{-11}$ W. Knowing that the car audio is 5 kms far from the transmitter, determine the power that needs to be transmitted using an antenna of gain $G_t = 3$ to supply the car audio with the necessary power.

R: Being $P_r = A_{eqr} \frac{PG_t}{4\pi r^2}$ we obtain $P = \frac{4\pi r^2 P_r}{A_{eqr} G_t} = \frac{4\pi 5000^2 \times 10^{-11}}{0.1 \times 3} = 10.4 \times 10^{-3}\,\text{W}$

It can be noticed that we obtain a very low power, as a matter of fact the transmitters use a power of roughly thousand watts to allow a reception in case of both outdoor obstacles and into premises.

Antenna reversibility

Each transmitting antenna can also work as a receiver, hence known the gain G of a transmitting antenna towards a determined direction, you can calculate its equivalent area A_{eq}, when the antenna receives from the same direction. It also proved that :

$$A_{eq} = \frac{\lambda^2}{4\pi} G$$

where λ is the wavelength used for the transmission. We would like to remind you that a radio transmission uses a determined frequency f to which corresponds a wavelength $\lambda = c/f$, where c represents the speed of light. Antenna designers, never provide G and A_{eq}, but just one of the two parameters (almost always the gain), being the two linked by the above mentioned relationship. Therefore if we know the receiving antenna gain G_r instead of the effective area, the last formula obtained in the previous paragraph turns into :

$$P_r = A_{eqr} \frac{PG_t}{4\pi r^2} = \frac{\lambda^2}{4\pi} G_r \frac{PG_t}{4\pi r^2} = P \left(\frac{\lambda}{4\pi r} \right)^2 G_t G_r$$

Example

Determine the effective area of an isotropic antenna at the frequency f_1 = 10 MHz and f_2 = 100 MHz.

R: At the frequency f_1 corresponds a wavelength $\lambda_1 = c/f_1 = 3\times10^8/10^7 = 30$ m.

At the frequency f_2 corresponds a wavelength $\lambda_2 = c/f_2 = 3\times10^8/10^8 = 3$ m.

Knowing that isotropic antennas have got a unitary gain (G = 1), you obtain:

$$A_{eq1} = \frac{\lambda_1^2}{4\pi} = \frac{30^2}{4\pi} = 71.6 \text{ m}^2 \text{ e } A_{eq2} = \frac{\lambda_2^2}{4\pi} = \frac{3^2}{4\pi} = 0.716 \text{ m}^2$$

It can be also noted that, at the same gain of an antenna, the effective area changes in an inversely proportional way to the frequency square. That means when using very high frequencies (as the ones relating to the microwave area, of about 10^{10} Hz) you need to arrange some antenna having a very high gain, and therefore very directional (as a very high gain corresponds to an antenna capability to concentrate the transmitted power towards a unique direction). For this reason microvawes radio links mostly occur between fixed locations (or between moving locations in very narrow environments, where also very small equivalent areas allow a good signal reception).

Power Flux Density and Fields

In the previous paragraphs, to analyse radio transmissions under a quantitative point of view, it has been used an energetic based approach, without taking into consideration that both the concurrent energies and powers are associated with electromagnetic waves. In a plain wave, the electric and magnetic fields vibrate orthogonally between them and in respect of the propagation direction. (Figure 3), and, as already stated, to a reasonable distance from the transmitting antenna, electromagnetic waves can be considered as plain waves.

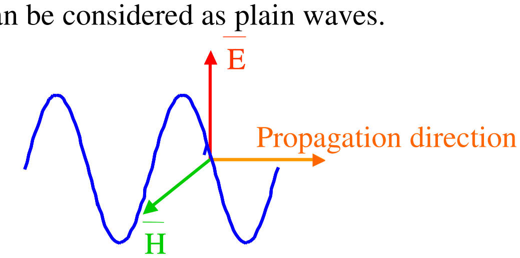

Figure 3. Orthogonality among electric field, magnetic field and motion direction in a plain wave.

The intensities of electric and magnetic fields vectors are function of time, respectively called E(t) and H(t), which in the easiest assumption are sinusoidal functions.

Considering the two functions effective values, E_{eff} e H_{eff}, we obtain that the power flux density is given by the product of the fields:

$$S = E_{eff} H_{eff}$$

Hence in a plain wave the two effective quantities are not independent, but are in correlation through: $\frac{E_{eff}}{H_{eff}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\Omega$

If the waves propagate in the vacuum (or in the atmosphere that shows electric permittivity and magnetic permeability almost equal to the vacuum one).

$\sqrt{\frac{\mu_0}{\varepsilon_0}}$ is also defined as vacuum impendency and it is indicated with R_0 or Z_0.

That is the reason why the relations $S = E_{eff}^2 / R_0 = H_{eff}^2 R_0$, which are very useful in electromagnetic fields, stand. In this context, some Italian Laws state that human body must not be exposed to a

power flux density higher than 1 W/m^2, which decrease to 0.1 W/m^2 in very crowded environments (such limits stand for frequencies in the range 3 MHz - 3 GHz).
To the first value of power flux density S_1 corresponds an effective electric field:

$$E_{eff\,1} = \sqrt{SR_0} = \sqrt{1\times 377} = 19.4 \text{ V/m}$$

To the second value of power density S_2 corresponds an effective electric field:

$$E_{eff\,2} = \sqrt{SR_0} = \sqrt{0.1\times 377} = 6.14 \text{ V/m}$$

The two obtained results are approximated with exposition limit parameters respectively of 20 V/m and 6 V/m.

Example

A base terrestrial radio station, for a mobile phone network, transmits a power of 50 W with an antenna whose gain is 30. Determine:

1- the effective electric field value 100 m far from the antenna along the maximum power flux direction.

2- at what distance you will have an effective field of a power flux equal to 20 V/m.

A1: the power flux density is: $S = \frac{PG_t}{4\pi r^2} = \frac{50\times 30}{4\pi 100^2} = 0.0119 \text{ W/m}^2$, to which corresponds an effective electric field: $E_{eff} = \sqrt{SR_0} = \sqrt{0.0119\times 377} = 2.12 \text{ V/m}$

A2: to obtain E_{eff1} = 20 V/m, the power flux density must be:
$S = E_{eff}^2 / R_0 = 20^2 / 377 = 1.06 \text{ W/m}^2$, from which we obtain:

$$r = \sqrt{\frac{PG_t}{4\pi S}} = \sqrt{\frac{50\times 30}{4\pi 1.06}} = 10.9 \text{ m}$$

Being the antennas of base terrestrial stations, almost next to a dimension of 1 m, the result is not precise, as at the obtained distance, the antenna is not likely to be considered dot-like. The electric field at such a distance is probably lower than 20 V/m, being the source less concentrated.

Transmission Systems and Real Reception

The formulations developed in the previous paragraphs, have taken into consideration the power transmitted by the transmitting antenna and the one received by the receiving antenna when the medium located between the two antenna doesn't register any absorbing power: using for instance, a hypothetical isotropic antenna, it has been assumed that the power spreads evenly in the space. If we consider a real link, between the transmitting and the receiving antenna, even if the straight line joining the two antennas were free, there would be some obstacles such a the underneath ground. These obstacles can give rise to some reflection and diffraction phenomena. In case of reflection, to the receiving antenna comes a straight wave, which combines with a reflected wave delayed, if compared with the first one, as it does a longer path (Figure 4).

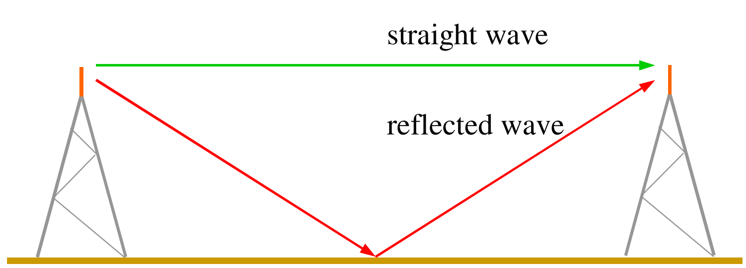

Figure 4. The straight wave combining with the ground reflected wave

Due to the delay, the two waves are to determine a phenomenon of constructive interference, where the received power is higher than the one it would be obtained without any reflection, or a destructive interference phenomenon, which gives to the received power a lower value than the one

it would be obtained without any reflection. The diffraction further spreads the energy in the space, thus reducing the received power, if compared with the one it would be obtained in totally free space conditions. Huygens principle states that a wave front is equal to an infinity of elementary sources, on which depends the electromagnetic field in the opposite space, - in respect of propagation direction, - the considered front. The presence of obstacles, which don't intercept the optic path, anyway perturbs the elementary sources, progressively weakening a wave energy. A usual cause of diffraction is represented by hills, mountains or artificial buildings, but also by the ground surface between the transmitter and the receiver. Telecommunication engineering has developed several models to quantify the reflection and diffraction effects, which can make the power received by the receiving antenna remarkably lower than the one expected in case of a free space. Satellite links extremely reduce the above phenomena.
We must also consider that radio communications apparatus are connected with respective antenna through cables (or waveguides). These media absorb (sometimes very significantly) part of the entry power (Figure 5).

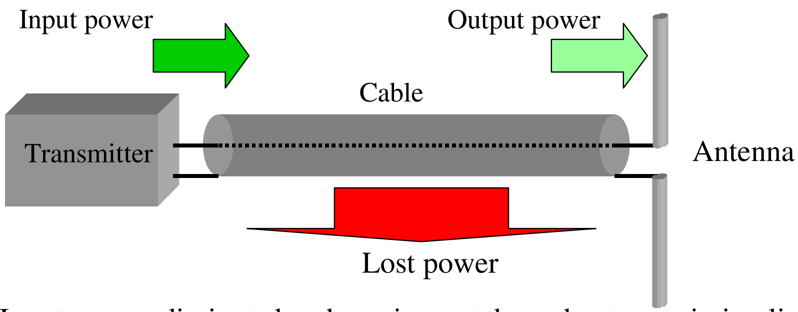

Figure 5. Input power, dissipated and coming out through a transmission line.

The ratio between the power given at the input of the cable and the coming out one is defined as cable loss. Therefore in a radio communications system if the following are defined:
-loss of cable connecting the transmitter to the transmitting antenna L_1
-loss of cable connecting the transmitter to the transmitting antenna $L2_1$
-distance between the two antennas r
-wavelength used λ (or frequency)
- transmitting antenna gain G_t
- receiving antenna gain G_r
In case of power P_{TX} given by the transmitter, in free space conditions we have:
-power supplied to the transmitting antenna: $P = \frac{P_{TX}}{L_1}$
-power flux density on receiving antenna: $S = \frac{PG_t}{4\pi r^2} = \frac{P_{TX}}{L_1}\frac{G_t}{4\pi r^2}$
-power received on the receiving antenna:

$$P_r = A_{eqr}S = A_{eqr}\frac{P_{TX}}{L_1}\frac{G_t}{4\pi r^2} = \frac{\lambda^2}{4\pi}G_r\frac{P_{TX}}{L_1}\frac{G_t}{4\pi r^2} = \frac{P_{TX}}{L_1}\left(\frac{\lambda}{4\pi r}\right)^2 G_t G_r$$

-power received by the receiver: $P_{RX} = \frac{P_r}{L_2} = P_{TX}\left(\frac{\lambda}{4\pi r}\right)^2\frac{G_t G_r}{L_1 L_2}$

In case of reflection and diffraction phenomena, the models used in Telecommunication Engineering supply an additional cable loss L_m so that:

$$P_{RX} = P_{TX}\left(\frac{\lambda}{4\pi r}\right)^2 \frac{G_t G_r}{L_1 L_2 L_m}$$

Example

Of a radio transmissions system the following data is known:

-transmitting power: P_{TX} = 1 mW

-gain of transmitting: G_t = 20

-receiving antenna equal to the transmitting

-loss of the cable connecting the transmitter to the transmitting antenna: L_1 = 1.52

-loss of the cable connecting the transmitter to the receiving antenna: L_2 = 2.63

-distance between the antennas: r = 980 m

-frequency used: f = 433 MHz

Determine the value of the received power supposing free space conditions occur, and the value of the additional loss L_m if $P_{RX1} = 10^{-10}$ W

A: the wavelength $\lambda = c/f = 3\times10^8/(433\times10^6) = 0.693$ m must be determined

Then, keeping in mind that in free space conditions the loss L_m has no influence ($L_m = 1$), we have:

$$P_{RX} = P_{TX}\left(\frac{\lambda}{4\pi r}\right)^2 \frac{G_t G_r}{L_1 L_2 L_m} = 10^{-3}\times\left(\frac{0.693}{4\pi 980}\right)^2 \frac{20\times 20}{1.52\times 2.63} = 3.17\times10^{-10}\ \text{W}$$

If the power received is P_{RX1}, we can calculate L_m from the previous formula, or simply notice that $L_m = P_{RX}/P_{RX1} = 3.17\times10^{-10}/10^{-10} = 3.17$.

It can be noticed how the power received is more than 100 times higher than the one needed by a mobile to censure a good reception. It's quite usual that a receiver receives powers far greater than the ones needed, which are generally safe. A transmitted power oversize is necessary to avoid high values in additional loss L_m , in case of obstacles, reflection and diffraction phenomena.

Exercises

1. How much power must an isotropic antenna transmit to produce, at a distance of 1 km, a power flux density of 10^{-10} W/m^2 ? And if the antenna gain is G = 50?
2. How much EIRP power must a transmitter installed on the moon, so that a receiver located on earth, of an effective area of 10 m^2 receive a power of 10 pW? (let's suppose Earth – Moon distance equal to 380000 kms).
3. With reference to the previous exercise, determine the power the antenna must transmit if it has a gain G = 3000 and, successively, an effective area of 5 m^2, at a frequency f_1 = 150 MHz e f_2 = 10000 MHz.
4. A radio transmission covering a distance of 55 kms must be set. The transmitting and receiving antenna gain are respectively G_t = 15 e G_r = 20; knowing that the cable connected with the transmitter has got a loss L_1 = 2 connected with the transmitter has got a loss L_2 = 1.5, calculate the power the transmitter must supply so as the received power is 10 pW, at a frequency of 450 MHz and considering an additional loss L_m = 10.
5. With reference to the previous exercise, determine the effective electric field 50 meters far from the transmitting antenna along the direction of maximum flux and the value the power given by the transmitter must assume so as, at such a distance, the effective field has a value of 6 V/m.

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